C. Phys Cumulative Exam
17 Feb 2010

A) $q_E = 2 + 1\, e^{-\beta\, 0.05\,eV}$

$$P(2) = 0.1 = \frac{1\, e^{-\beta \epsilon_2}}{2 + e^{-\beta \epsilon_2}} \rightarrow \quad 0.2 + 0.1\, e^{-\beta\epsilon_2} = e^{-\beta\epsilon_2}$$

$$0.2 = 0.9\, e^{-\beta\epsilon_2}$$

$$\ln \frac{0.2}{0.9} = -\beta \epsilon_2$$

$$k_B T = \beta = \frac{1}{\epsilon_2} \ln \frac{0.9}{0.2}$$

$$T = \frac{1}{k_B \epsilon_2} \ln \frac{0.9}{0.2}$$

$$T = \left(\frac{1}{\frac{1.38 \times 10^{-23}\, J/K}{1.602 \times 10^{-19}\, J/eV}\; 0.05\, eV} \right) \ln \frac{0.9}{0.2}$$

$$T = 3.5 \times 10^5\ K$$

B) $\langle E \rangle = k_B T^2 \left(\frac{\partial}{\partial T} \ln Q \right)_{N,V}$

$$\ln Q = N \ln (V - b) - \ln N! + \frac{3N}{2} \ln \left(\frac{2\pi m k_B}{h^2} \right) + \frac{3N}{2} \ln T$$

T in only last term

(a) $\langle E \rangle = k_B T^2 \frac{\partial}{\partial T} \left(\frac{3N}{2} \ln T \right)_{N,V}$

$$= \frac{3N}{2} k_B T^2 \left(\frac{1}{T} \right) = \frac{3}{2} N k_B T = \frac{3}{2} n R T$$

$N_A n = N$, $N_A k_B = R$

(b) $\langle P \rangle = k_B T \left(\frac{\partial}{\partial V} \ln Q\right)_{T,N}$

$\ln Q = N \ln(V-b) - \ln N! + \frac{3}{2} N \ln\left(\frac{2\pi m k_B}{h^2}\right) + \frac{3}{2} N \ln T$

V is only in first term

$\langle P \rangle = k_B T \frac{\partial}{\partial V}\left(N \ln(V-b)\right)_{T,N}$

$\langle P \rangle = N k_B T \frac{\partial}{\partial V} \ln(V-b) = \frac{N k_B T}{(V-b)} = \frac{nRT}{(V-b)}$

C) $A = -k_B T \ln Q$ (Helmholtz Free energy)

use $q_{TRANS} = \left(\frac{2\pi m k_B T}{h^2}\right)^{3/2} V$ only degree of freedom for He, Ne

$A = -k_B T \ln q_{TRANS} = -k_B T \left[\frac{3}{2} \ln\left(\frac{2\pi m}{h^2} k_B T\right) + \ln V\right]$

so $A \propto \ln(m)$ all other things being equal.

A for Ne > A for He

D) $q_{elec} = g_1 + g_2 e^{-\beta E_2} + g_3 e^{-\beta E_3} \ldots$

$\beta = \frac{1}{k_B T}$ as $T \to \phi$ $\beta \to \infty$

Thus all higher terms $e^{-\beta \epsilon}$ go to zero as $T \to 0$

$q_{elec}(T \to 0) = g_1$

$N = 4$ atoms $\qquad$ $N = 2$ atoms

E) NH_3 vs HF

3 rotational modes $\qquad$ 2 rotational modes

$3N - 3 = 9$ vibrational modes $\qquad$ 1 vibrational mode

C_V of NH_3 should be much larger than that of HF

F) Ignoring the mass difference the spring constant for the triple bond should be much larger than the spring constant for the double bond

$$\Theta_{VIB} = \frac{h\nu}{k_B} \quad , \quad \nu = \frac{1}{2\pi}\sqrt{\frac{k}{m_{eff}}}$$

thus $\Theta_{VIB} \propto \sqrt{\text{Spring Const}}$

G) $I = 1\hbar$ has three states $m = +1, \phi, -1$

$$q_{magnetic} = e^{+1\,\gamma\mu_N B} + e^{0\,\gamma\mu_N B} + e^{-1\,\gamma\mu_N B}$$

$$q_{magnetic} = 1 + e^{-\gamma\mu_N B} + e^{+\gamma\mu_N B}$$