This exam focuses on electromagnetic waves. The exam will be graded out of 100 points, with the distribution indicated at the start of each question. You can refer to the attached equation sheet to find constants and other equations that may prove useful in addressing the questions below.

I. (10 points) The Basics
   a. Give the upper and lower wavelength limits (in nm) of the visible region of the electromagnetic spectrum.
   b. What fraction of the electromagnetic spectrum lies in the visible range?
   c. Determine the frequency of x-rays with wavelength $0.067 \times 10^{-15}$ m moving in vacuum.
   d. What is the momentum of the photons described in I.c.?
   e. A VLF (very low frequency) radio wave has a frequency of only 30 Hz. At what speed does it travel in free space?

II. (20 points) Maxwell’s Equations
   a. Calculate the speed of light from the relation $c = \sqrt{\frac{1}{\varepsilon_0 \mu_0}}$ and show that the equation is dimensionally correct.
   b. The four basic equations of electromagnetism (Maxwell’s equations) are given on the attached equation sheet. Associate each equation with the four basic physics laws listed below, and give a short description of what each equation physically describes
      i. Ampère’s law
      ii. Faraday’s law of induction
      iii. Gauss’s law for electricity
      iv. Gauss’s law for magnetism
   c. A parallel-plate capacitor with circular plates of radius R is charged. Derive an expression for the induced magnetic field at position r, where r < R.
   d. The induced magnetic field for the same parallel-plate capacitor in II.c., when r = R, is derived to be $B(r) = \frac{1}{2} \mu_0 \varepsilon_0 R \frac{dE}{dt}$. For $dE/dt = 1.0 \times 10^{12}$ V/m·s and R = 50 mm, find B(R).
   e. Why are magnetic effects of conduction currents in wires so easy to detect, but the magnetic effects of displacement currents in capacitors so hard to detect?

III. (20 points) Traveling Waves and Maxwell’s Equations
   a. Sketch on a Cartesian axis the electric and magnetic fields associated with plane-polarized electromagnetic radiation propagating in the +x direction
   b. What is the difference between plane-polarized and circularly-polarized electromagnetic radiation?
   c. Maxwell’s relation $\int \mathbf{E} \cdot d\ell = -\frac{d\Phi_B}{dt}$ applied to plane waves leads to the following equivalence: $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$. Show that the speed of electromagnetic radiation in vacuum is the ratio of amplitudes of the electric and magnetic components of the plane wave.
d. Maxwell’s relation \( \oint \mathbf{B} \cdot d\ell = \mu_0 \varepsilon_0 \frac{d\Phi}{dt} + \mu_0 j \) applied to plane waves leads to the following equivalence: \( -\frac{\partial B}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \), where the conduction current is taken to be zero. Show that the speed of electromagnetic radiation in vacuum is \( c = \sqrt{\varepsilon_0 \mu_0} \).

e. Prove that, for any point in an electromagnetic wave, the time-averaged density of the energy stored in the electric field equals that of the energy stored in the magnetic field.

IV. (20 points) *Energy Transport and Radiation Pressure*

a. An electromagnetic wave is traveling in the negative y-direction. At a particular position and time, the electric field is along the positive z-direction and has magnitude 100 V/m. What are the direction and magnitude of the magnetic field at that position and at that time?

b. Show that in a plane-polarized electromagnetic wave the average intensity is given by
   \[ S = \frac{E_m^2}{2\mu_0 c} \]

c. A helium-neon laser has a continuous power output of 5.0 mW at 633 nm. The beam is focused by a lens to a circular spot whose effective diameter is 2.0 wavelengths. Determine the intensity of the focused beam and the radiation pressure exerted on a perfectly reflecting sphere whose diameter is that of the focal spot.

d. Prove that for a plane wave at normal incidence on a plane surface the radiation pressure on the surface is equal to the energy density in the beam outside of the surface. Note that this relation holds no matter what fraction of the incident energy is reflected.

V. (20 points) *Doppler Effect*

a. A rocket ship is receding from the earth at a velocity \( \beta = 0.20 \). A light in the ship appears orange to the passengers on the ship. Would the color appear to be blue or red to an observer on the earth? Briefly explain your answer.

b. Could you go through a red light fast enough to have it appear green? Take \( \lambda = 620 \text{ nm} \) for red light, \( \lambda = 540 \text{ nm} \) for green light to determine the required velocity.

c. Show that for slow speeds, the Doppler shift can be written in the approximate form
   \[ \frac{\Delta \lambda}{\lambda} = \frac{v}{c}, \]
   where \( v \) is the speed and \( \Delta \lambda \) the change in wavelength.

d. Microwaves are reflected from a distant airplane approaching the wave source. It is found that when the reflected waves are beat against the waves radiating from the source, the beat frequency is 990 Hz. If the microwaves are 0.10 m in wavelength, what is the approaching speed of the plane?

VI. (10 points) *Plane Waves Through Media*

a. Why does the speed of blue light differ from that of red light in fused quartz?

b. The wavelength of yellow sodium light in air is 589 nm.
   i. What is its frequency?
   ii. What is the wavelength in glass whose index of refraction is 1.52?
   iii. What is the speed of light in this glass
\( c = \lambda v \quad \text{and} \quad c = 2.997 \times 10^8 \text{ m/s} \)

\( c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \quad \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2 \quad \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \)

\( \lambda = \frac{h}{p} \quad h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} \quad p = mv \)

\( K.E. = \frac{1}{2} mv^2 \quad K.E. = \frac{p^2}{2m} \quad U = K.E. + P.E. \)

\( T = \frac{Wb}{m^2} = \frac{Vs}{m^2} \quad V = \frac{W}{A} \quad W = \frac{J}{s} \quad J = \frac{kgm^2}{s^2} = \text{Nm} \)

\( C = A \cdot s \quad P = \frac{F}{A} \)

Maxwell’s Equations:

\[ \oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0} \quad \oint \mathbf{B} \cdot d\mathbf{S} = 0 \quad \oint \mathbf{E} \cdot d\ell = -\frac{d\Phi_B}{dt} \quad \oint \mathbf{B} \cdot d\ell = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i \]

Plane Wave formulation:

\( E(x,t) = E_m \sin(kx - \omega t) \quad B(x,t) = B_m \sin(kx - \omega t) \)

Other equations:

\( F_E = q_0 \mathbf{E} \quad F_B = q_0 \mathbf{v} \times \mathbf{B} \)

\( \rho_E = \frac{1}{2} \varepsilon_0 E^2 \quad \rho_B = \frac{1}{2} \frac{B^2}{\mu_0} \quad \phi_E = \oint \mathbf{E} \cdot d\mathbf{S} \quad \phi_B = \oint \mathbf{B} \cdot d\mathbf{S} \)

\( E = \frac{q}{\varepsilon_0 A} \quad \mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \quad S = \frac{1}{\mu_0} \mathbf{E} \mathbf{B} \quad \mathbf{S} = \left( \frac{1}{\mu_0} \mathbf{E} \mathbf{B} \right) \)

\( E = cB \quad dU = dU_E + dU_B \quad p = \frac{U}{c} \quad \Delta F = p / \Delta t \)

\( I = I_o \cos^2 \theta \quad \nu' = \nu - \frac{1 - \beta}{\sqrt{1 - \beta^2}} \quad \beta = v / c \)

\( n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \nu_n = c / n \quad \lambda_n = \lambda / n \quad \sin \theta_n = n_2 / n_1 \)

For closed loop with radius \( r \):

\[ \oint \mathbf{B} \cdot d\ell = \oint B \cos \theta \cdot d\ell = B \oint d\ell = B(2\pi r) \]