1. (20 points) Consider a hydrogen atom with the nucleus at the origin and the electron in a $|2p_z\rangle$ state.
   
   a. Derive an expression for the $zz$ component of the atomic quadrupole moment. The quadrupole operator is $-Ar^2Y_2^0(\theta, \phi)$ where $A$ is a constant. Simplify the result as much as you can (do the integrations).
   
   b. Derive an expression for the $zz$ component of the electric field gradient at the nucleus. The electric field gradient operator is $\frac{B}{r^3}Y_2^0(\theta, \phi)$ where $B$ is a constant. Simplify the result as much as you can (do the integrations).
   
2. (30 points) Consider the $HF$ molecule in its ground state.
   
   a. How would you measure its dipole moment?
   
   b. How would you calculate its dipole moment?
   
   c. Sketch the anticipated dipole moment as a function of bond length.
   
3. (10 points) How would you calculate the dipole polarizability of the H atom in its ground state?
   
4. (20 points) Identify which of the following molecules is paramagnetic and explain your answer.
   
   a. $O_2$
   
   b. ON
   
   c. $N_2$
   
   d. HOO
   
   e. HNO
   
5. (20 points) The spin Hamiltonian for the H atom in its ground state has the form
   
   $\hat{H} = g_e \beta_e \hat{H}_z + g_N \beta_N \hat{H}_z + a\hat{\mathbf{\sigma}} \cdot \hat{\mathbf{S}}$
   
   a. Identify the terms.
   
   b. Calculate, to first order, the energy levels of this system.
Useful equations

\[ |2p_z| = \sqrt{\frac{1}{24a_0^5} r \exp\left(-\frac{r}{2a_0}\right)} Y_0^0(\theta, \phi) \]

\[ Y_1^0(\theta, \phi) Y_1^0(\theta, \phi) = \frac{1}{\sqrt{5\pi}} Y_2^0(\theta, \phi) + \frac{1}{\sqrt{4\pi}} Y_0^0(\theta, \phi) \]

\[ \int_0^\infty x^N e^{-ax} dx = \frac{N!}{a^{N+1}} \]

\[ \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi \ Y_{l_1}^{m_1}(\theta, \phi) Y_{l_2}^{m_2}(\theta, \phi) = \delta_{l_1 l_2} \delta_{m_1 m_2} \]

\[ \hat{S}_x \alpha = \frac{\beta}{2} \quad ; \quad \hat{S}_x \beta = \frac{\alpha}{2} \]

\[ \hat{S}_y \alpha = \frac{i\beta}{2} \quad ; \quad \hat{S}_y \beta = \frac{-i\alpha}{2} \]