This exam deals with aspects regarding classical kinematics applied to elastic and inelastic collisions between bodies in one and two dimensions. The exam will be graded out of 100 points, with the distribution indicated at the start of each question.

I. (5 points). State Newton’s three laws of motion.

II. (15 points). Elastic Collisions in One Dimension
a. Using conservation laws, show for a one-dimensional elastic collision that the relative velocity of approach before collision is equal to the relative velocity of separation after collision.
b. Determine the fraction of kinetic energy lost by a neutron, which undergoes a head-on collision with a lead-208 \((^{208}\text{Pb})\) atomic nucleus and a hydrogen-1 \((^1\text{H})\) atomic nucleus.

\[
\begin{align*}
M(\text{n}) & = 1.008665 \text{ amu} \\
M(^1\text{H}) & = 1.00782 \text{ amu} \\
M(\text{Pb}) & = 207.9766 \text{ amu} \\
1 \text{ amu} & = 1.660 \times 10^{-27} \text{ kg}
\end{align*}
\]
c. Compare the results obtained for each atomic nucleus in (II.b) and explain why water is used in nuclear reactors to slow down the fast neutrons produced during the fission process in order to sustain the chain reaction.

III. (10 points) Inelastic Collisions
a. A deuteron (hydrogen-2) particle is accelerated by a cyclotron to a speed of \(1.0 \times 10^7 \text{ m/s}\), and collides with another deuteron at rest. If the two particles stick together head-on to form a helium-4 nucleus, find the speed of the resulting nucleus.

\[
\begin{align*}
M(^2\text{H}) & = 2.0141 \text{ amu} \\
M(^4\text{He}) & = 4.0026 \text{ amu} \\
1 \text{ amu} & = 1.660 \times 10^{-27} \text{ kg}
\end{align*}
\]
b. Is energy conserved in the collision defined in (III.a)? Provide a brief explanation for your answer.

IV. (30 points) Collisions in Two Dimensions
A fast-moving proton with velocity \(v_{1i}\) collides elastically with another proton at rest. The original proton is scattered at some angle \(\theta_{1f}\) from its initial direction with final velocity \(v_{1f}\), while the other proton moves in a direction with velocity \(v_{2f}\) after collision making an angle \(\theta_{2f}\) with the initial direction of the incident proton.

a. Make a sketch of the two-dimensional scattering plane, indicating the initial and final velocity vectors, as well as the scattering angles \(\theta_{1f}\) and \(\theta_{2f}\).
b. Give the two scalar equations for the x- and y-component of motion based on conservation of linear momentum.
c. If the initial conditions of the scattering experiment are known (proton mass and incident velocity \(v_{1i}\)), how many parameters are left to be determined based on the two scalar equations?
d. If the initial proton velocity \(v_{1i} = 500 \text{ m/s}\), and the original proton is scattered at \(\theta_{1f} = 60^\circ\) from its initial direction, then

i. What are the speeds of the two protons after the collision?
ii. What is the direction of the velocity of the target proton after the collision?

\[
\begin{align*}
M(\text{p}) & = 1.00727 \text{ amu}, \ 1 \text{ amu} = 1.660 \times 10^{-27} \text{ kg}
\end{align*}
\]
V. (20 points) **Compton effect**

a. The Compton effect related the interaction of a photon with an electron that can be considered at rest. Using classical kinematics (ignoring relativistic effects), show that the shift in the wavelength of the scattered photon $\Delta \lambda$ can be expressed by the simple relation

$$\Delta \lambda = \lambda' - \lambda = \frac{h}{m_0 c} \left( 1 - \cos \phi \right)$$

where $\lambda$ is the initial photon wavelength, $\lambda'$ is the outgoing photon wavelength, $h$ is Planck’s constant, $m_0$ is the electron rest mass, $c$ is the velocity of light, and $\phi$ is the angle of the scattered photon relative to the initial photon direction.

b. For what reaction condition(s) will the value of $\Delta \lambda$ be maximum?

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VI. (20 points) **Center-of-mass reference frame**

A two-body reaction, resulting in the production of two final products, is more conveniently analyzed in the center-of-mass reference frame in which the total linear momentum of the reacting particles is zero.

a. Consider the generic reaction

$$a + X \rightarrow Y + b$$

where $a$ is the projectile traveling with total kinetic energy $T_a$ and the target particles $X$ are at rest in the laboratory. Derive an expression for the initial center-of-mass velocity $v_{cm,i}$.

b. Since the total linear momentum in the center-of-mass reference frame must be zero, what can be said about the relative direction of travel between products $Y$ and $b$ in the center of mass reference frame?

c. For elastic collisions, show that the center-of-mass velocity for the initial reaction conditions $v_{cm,i}$ is equal to the post-collision center-of-mass velocity $v_{cm,f}$.

d. Consider the reaction of an oxygen-16 ($^{16}$O) nucleus with a nickel-64 ($^{64}$Ni) nucleus at a center-of-mass energy of 48 MeV. In the laboratory reference frame, where the $^{64}$Ni nucleus is at rest, determine the kinetic energy of the $^{16}$O nucleus.

\[
M(^{16}\text{O}) = 15.9949 \text{ amu} \\
M(^{64}\text{Ni}) = 63.9280 \text{ amu} \\
1 \text{ amu} = 1.660 \times 10^{-27} \text{ kg}
\]