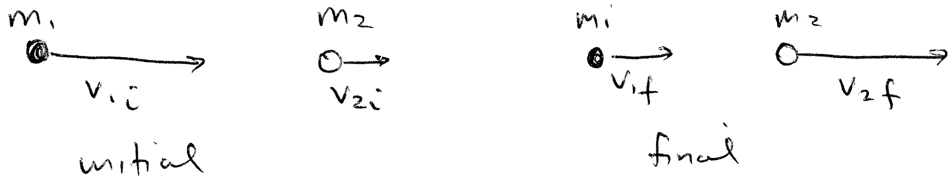
I

1. Law of inertia – if no net force acts on a body, its acceleration is zero

2. $F = ma$ acceleration is directly proportional to net force applied, and inversely proportional to mass of the body

3. For every action, there is an equal and opposite reaction

II a.



for conservation of momentum.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

also, for elastic collision, energy is conserved.

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

rewriting each equation's by gathering terms with like masses together:

$$m_1 (v_{1i} - v_{1f}) = m_2 (v_{2f} - v_{2i})$$

$$m_1 (v_{1i}^2 - v_{1f}^2) = m_2 (v_{2f}^2 - v_{2i}^2)$$

divide the first relation by the second:

$$\frac{\cancel{(v_{1i} - v_{1f})}}{\cancel{(v_{1i} - v_{1f})}(v_{1i} + v_{1f})} = \frac{\cancel{(v_{2f} - v_{2i})}}{\cancel{(v_{2f} - v_{2i})}(v_{2f} + v_{2i})}$$

$$(v_{1i} + v_{1f}) = (v_{2f} + v_{2i})$$

and gather initial + final velocities together

$$\boxed{(v_{1i} - v_{2i}) = (v_{2f} - v_{1f})}$$

II b.

		momentum	kinetic energy
● → (H)	initial	$m_n V_{ni}$	$\frac{1}{2} m_n V_{ni}^2$
	final	$m_n V_{nf} + m_H V_{Hf}$	$\frac{1}{2} m_n V_{nf}^2 + \frac{1}{2} m_H V_{Hf}^2$

the fractional decrease in neutron kinetic energy is.

$$\frac{K_i - K_f}{K_i} = \frac{\cancel{\tfrac{1}{2}}\cancel{m_n} V_{ni}^2 - \cancel{\tfrac{1}{2}}\cancel{m_n} V_{nf}^2}{\cancel{\tfrac{1}{2}}\cancel{m_n} V_{ni}^2} = \frac{V_{ni}^2 - V_{nf}^2}{V_{ni}^2} = 1 - \frac{V_{nf}^2}{V_{ni}^2}$$

can express V_{nf} in terms of V_{ni} using conservation laws.

from IIa. we know that $(V_{1i} - V_{2i}) = (V_{2f} - V_{1f})$

for this case $V_{ni} = V_{Hf} - V_{nf}$ (since $V_{Hi} = 0$ @ rest)

Putting this relation into the momentum conservation law

$$m_n V_{ni} = m_n V_{nf} + m_H V_{Hf}$$

$$m_n V_{ni} = m_n V_{nf} + m_H (V_{nf} + V_{ni})$$

$$= m_n V_{nf} + m_H V_{nf} + m_H V_{ni}$$

$$m_n V_{ni} - m_H V_{ni} = m_n V_{nf} + m_H V_{nf}$$

$$V_{nf} = \left(\frac{m_n - m_H}{m_n + m_H}\right) V_{ni}$$

$$\frac{K_i - K_f}{K_i} = 1 - \frac{V_{nf}^2}{V_{ni}^2} = 1 - \frac{\left(\frac{m_n - m_H}{m_n + m_H}\right)^2 \cancel{V_{ni}^2}}{\cancel{V_{ni}^2}} = 1 - \left(\frac{m_n - m_H}{m_n + m_H}\right)^2$$

$$= 1 - \left(\frac{1.008665\,\text{amu} - 1.007825\,\text{amu}}{1.008665\,\text{amu} + 1.007825\,\text{amu}}\right)^2$$

$$= 1 - \left(\frac{0.000845\,\text{amu}}{2.016485\,\text{amu}}\right)^2$$

$$\sim 1 = \boxed{100\%}$$

For Pb

$$\frac{K_i - K_f}{K_i} = 1 - \left(\frac{1.008665\,\text{amu} - 207.9766\,\text{amu}}{1.008665\,\text{amu} + 207.9766\,\text{amu}}\right)^2 = 1 - \left(\frac{-206.968}{208.9852}\right)^2$$

II c Water contains significant portion of H atoms, which are very effective in slowing down neutrons. The fractional K.E. lost by a neutron (fast moving) to a hydrogen atom is essentially 100%.

III a.

$$m_{1i}({}^2H)\, v_{1i}({}^2H) + m_{2i} v_{2i} = m_{3f} v_{3f}$$

since one deuteron is @ rest

$$m_{1i} v_{1i} = m_3 v_{3f}$$

$$(2.0144 \text{ amu})(1.0\times10^{7} \text{ m/s}) = (4.0026 \text{ amu})(v_{3f})$$

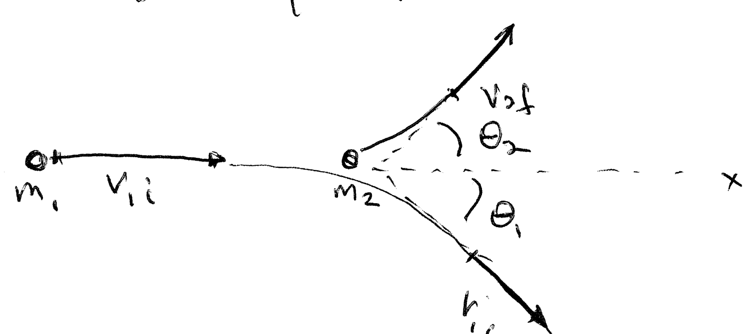
$$v_{3f} = \left(\frac{2.0144 \text{ amu}}{4.0026 \text{ amu}}\right)(1.0\times10^{7} \text{ m/s}) = \boxed{5.0\times10^{6} \text{ m/s}}$$

b. ~~No,~~ Yes total energy is ~~not~~ conserved. Collision is inelastic so, kinetic energy is <u>not</u> conserved. Goes into Q value for reaction. Can check by

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_3 v_{3f}^2$$

due to mass difference between d + d and ⁴He, right and left hand sides of equation will not be equal.

IV a.



b. x component. $\boxed{m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2}$

y component $\boxed{\emptyset = m_2 v_{2f} \sin\theta_2 - m_1 v_{1f} \sin\theta_1}$

IV c. Still have four unknowns θ_1, θ_2, V_{1f}, and V_{2f}. Need at least 2 more pieces of information to solve problem... usually conserved kinetic energy (for elastic collisions) and one final reaction parameter (either angle or velocity)

d. Since $m_1 = m_2$

$$\left.\begin{aligned} V_{1i} &= V_{1f}\cos\theta_1 + V_{2f}\cos\theta_2 \\ V_{1f}\sin\theta_1 &= V_{2f}\sin\theta_2 \end{aligned}\right\}\text{ conservation of momentum}$$

$$V_{1i}^2 = V_{2f}^2 + V_{1f}^2 \quad \}\text{ conservation of kinetic energy.}$$

by manipulating these equations:

$$V_{1f} = V_{1i}\cos\theta_1 = (500\,m/s)(\cos 60^\circ) = \boxed{250\,m/s}$$

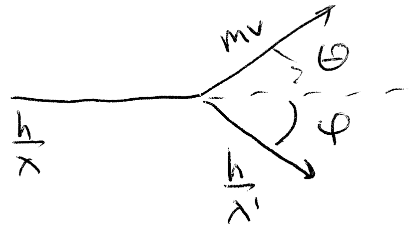
$$V_{2f} = \sqrt{V_{1i}^2 - V_{1f}^2} = \sqrt{(500\,m/s)^2 - (250\,m/s)^2} = \sqrt{187{,}500\,m^2/s^2} = \boxed{433\,m/s}$$

$$\sin\theta_2 = \frac{V_{1f}}{V_{2f}}\sin\theta_1$$

$$\sin\theta_2 = \frac{250\,m/s}{433\,m/s}\sin 60^\circ = (0.577)(0.8660) = 0.50$$

$$\boxed{\theta_2 = 30^\circ}$$

(v) a.



momentum:

x-component $\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\varphi + mv\sin\theta$

y-component $0 = mv\sin\theta - \frac{h}{\lambda'}\sin\varphi$

energy: $\frac{hc}{\lambda} = \frac{hc}{\lambda'} + \frac{1}{2}mv^2$

To eliminate θ, square and add first equations

$$\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\varphi\right)^2 = (mv\cos\theta)^2$$

$$\left(\frac{h}{\lambda}\right)^2 - \frac{2h^2}{\lambda\lambda'}\cos\varphi + \left(\frac{h}{\lambda'}\right)^2\cos^2\varphi = m^2v^2\cos^2\theta$$

$$+\ (mv\sin\theta)^2 = \left(\frac{h}{\lambda'}\sin\phi\right)^2$$

$$\left(\frac{h}{\lambda}\right)^2 - \frac{2h^2}{\lambda\lambda'}\cos\varphi + \left(\frac{h}{\lambda'}\right)^2\cos^2\varphi + \left(\frac{h}{\lambda'}\right)^2\sin^2\varphi = m^2v^2\cos^2\theta + m^2v^2\sin^2\theta$$

$\cos^2\varphi + \sin^2\varphi = 1$ $\qquad$ $\cos^2\theta + \sin^2\theta = 1$

$$\left(\frac{h}{\lambda}\right)^2 - \frac{2h^2}{\lambda\lambda'}\cos\varphi + \left(\frac{h}{\lambda'}\right)^2 = m^2v^2$$

from energy $m^2v^2 = 2mhc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$

$$\left(\frac{h}{\lambda}\right)^2 - \frac{2h^2}{\lambda\lambda'}\cos\varphi + \left(\frac{h}{\lambda'}\right)^2 = 2mhc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

multiply both sides by $\lambda\lambda'$, pull out h^2 from left

$$h^2\left(\frac{\lambda'}{\lambda} - 2\cos\varphi + \frac{\lambda}{\lambda'}\right) = 2mhc(\lambda' - \lambda) \qquad \lambda' - \lambda = \Delta\lambda$$

$$\Delta\lambda = \frac{h}{2mc}\left(\frac{\lambda'}{\lambda} + \frac{\lambda}{\lambda'} - 2\cos\varphi\right)$$

when $\Delta\lambda$ small, $\frac{\lambda'}{\lambda} + \frac{\lambda}{\lambda'} \sim 2$

$$\Delta\lambda = \frac{h}{2mc}(2 - 2\cos\varphi) = \boxed{\frac{h}{\cancel{2}mc}(1 - \cos\varphi)}$$

b. $\Delta\lambda$ maximum when $\cos\varphi = -1$ $\quad$ $\varphi = 180°$

when photon is backscattered to incident photon direction.

VI

$a + X \rightarrow Y + b$

a.

lab frame $p = m_a v_a$

~~Com~~ CM frame $p = 0 = m_a v_a' + m_X v_X'$ where primes indicate center of mass coordinate

$v_a' = v_a - v_{cm}$ where v_{cm} is center of mass velocity

$v_X' = -v_{cm}$

using CM frame $m_a v_a' = -m_X v_X'$, and by substitution

$m_a(v_a - v_{cm}) = -m_X(-v_{cm})$

$m_a v_a - m_a v_{cm} = +m_X v_{cm}$

$m_a v_a = m_a v_{cm} + m_X v_{cm}$ $\boxed{v_{cm} = \left(\frac{m_a}{m_a + m_X}\right) v_a}$

b. for total momentum to be zero, particles must travel @ 180° in CM reference frame

c. For center of mass frame

$m_b v_b' + m_Y v_Y' = 0$ and $v_b' = v_b - v_{cm,f}$

$v_Y' = v_Y - v_{cmf}$

$m_b(v_b - v_{cm,f}) + m_Y(v_Y - v_{cm,f}) = 0$

$m_b \cancel{v_b} - m_b v_{cmf} + m_Y v_Y - m_Y v_{cmf} = 0$

$m_b v_b + m_Y v_Y = m_b v_{cmf} + m_Y v_{cmf}$

$\frac{m_b v_b + m_Y v_Y}{(m_b + m_Y)} = v_{cm,f} = \frac{m_a v_a}{(m_b + m_Y)}$

assuming elastic ~~collision~~ collision $m_b + m_Y = m_a + m_X$

$v_{cm,f} = \frac{m_a v_a}{(m_a + m_X)} = v_{cm,i}$

d.

$T_{cm} = 48 MeV$

$$T_p = T_{cm}\left(\frac{m_T + m_p}{m_p}\right)$$

T_p = K.E. projectile in lab frame

m_p = mass of projectile

m_T = mass of target

$$T_p = 48 MeV \left(\frac{15.9949\,amu + 63.9280\,amu}{15.9949\,amu}\right) = \boxed{240\ MeV}$$