1. Law of inertia - if no force acts on a body, its acceleration is zero.

2. Newton's law of acceleration is directly proportional to net force applied, and inversely proportional to mass of the body.

3. For every action, there is an equal and opposite reaction.

\[ m_1 v_{1\text{c}} + m_2 v_{2\text{c}} = m_1 v_{1\text{f}} + m_2 v_{2\text{f}} \]

Also, for elastic collision, energy is conserved.

\[ \frac{1}{2} m_1 v_{1\text{c}}^2 + \frac{1}{2} m_2 v_{2\text{c}}^2 = \frac{1}{2} m_1 v_{1\text{f}}^2 + \frac{1}{2} m_2 v_{2\text{f}}^2 \]

Rewriting each equation's by gathering terms with like masses together:

\[ m_1 (v_{1\text{c}} - v_{1\text{f}}) = m_2 (v_{2\text{f}} - v_{2\text{c}}) \]

\[ m_1 (v_{1\text{c}}^2 - v_{1\text{f}}^2) = m_2 (v_{2\text{f}}^2 - v_{2\text{c}}^2) \]

Divide the first relation by the second:

\[ \frac{(v_{1\text{f}} - v_{1\text{c}})}{(v_{1\text{c}} - v_{1\text{f}})(v_{1\text{c}} + v_{1\text{f}})} = \frac{(v_{2\text{f}} - v_{2\text{c}})}{(v_{2\text{f}} + v_{2\text{c}})(v_{2\text{f}} + v_{2\text{c}})} \]

\[ (v_{1\text{c}} - v_{1\text{f}}) = (v_{2\text{f}} + v_{2\text{c}}) \]

and gather initial and final velocities together:

\[ (v_{1\text{c}} - v_{2\text{c}}) = (v_{2\text{f}} - v_{1\text{f}}) \]
the fractional decrease in neutrons kinetic energy is

\[
\frac{K_i - K_f}{K_i} = 1 - \left( \frac{V_{n_f}^2}{V_{n_i}^2} \right) = 1 - \left( \frac{\frac{m_n V_{n_i}^2}{m_n V_{n_i}^2}}{\frac{m_n V_{n_i}^2}{m_n V_{n_i}^2}} \right) = 1 - \left( \frac{V_{n_i}^2}{V_{n_i}^2} \right) = 1 - \left( \frac{V_{n_i}^2}{V_{n_i}^2} \right)
\]

can express \( V_{n_f} \) in terms of \( V_{n_i} \) using conservation laws.

from \( \Pi_a \) we know that \( (V_{1i} - V_{2i}) = (V_{2f} - V_{4f}) \)

for this case \( V_{n_i} = V_{4f} \) or \( V_{2f} \) (since \( V_{3f} = 0 \) e rest)

Putting this relation into the momentum conservation law

\[
m_n V_{n_i} = m_n V_{n_f} + m_H V_{H_f}
\]

\[
m_n V_{n_i} = m_n V_{n_f} + m_H (V_{H_f} + V_{n_i})
\]

\[
= m_n V_{n_f} + m_H V_{n_f} + m_H V_{n_i}
\]

\[
m_n V_{n_i} + m_H V_{n_i} = m_n V_{n_f} + m_H V_{n_f}
\]

\[
V_{n_f} = \left( \frac{m_n + m_H}{m_n + m_H} \right) V_{n_i}
\]

\[
\frac{K_i - K_f}{K_i} = \frac{\left( \frac{m_n - m_H}{m_n + m_H} \right)^2}{V_{n_i}^2} = 1 - \left( \frac{m_n - m_H}{m_n + m_H} \right)^2
\]

\[
\approx 1 - \left( \frac{0.000845\text{amu}}{2.089852\text{amu}} \right)^2
\]

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\]

For Pb

\[
\frac{K_i - K_f}{K_i} = 1 - \left( \frac{1.008665\text{amu} - 207.9766\text{amu}}{1.008665\text{amu} + 207.9766\text{amu}} \right)^2
\]

\[
\approx 1 - \left( \frac{-206.968}{208.9852} \right)^2
\]

\[
\approx 1 - \left( \frac{-206.968}{208.9852} \right)^2
\]
IIc. Water contains significant part of H atoms, which are very effective in slowing down neutrons. The fractional k.e. lost by a neutron (fast moving) to a hydrogen atom is essentially 100%. 

IIIa. \[ m_i v_{ic} + m_2 v_{ic} = m_3 v_{fc} \]

Since one deuteron is @ rest
\[ m_1 v_{ic} = m_3 v_{fc} \]

\[ (2.014 u amu) \times (1.0 \times 10^7 m/s) = (4.0026 u amu) \times (V_{fc}) \]

\[ V_{fc} = \frac{(2.014 u amu) \times (1.0 \times 10^7 m/s)}{(4.0026 u amu)} = \frac{5.0 \times 10^6 m/s}{1} \]

b. Yes total nuclear energy is conserved. Collision is inelastic so kinetic energy is not conserved. Goes into Q value for reaction. Can check by:
\[ \frac{1}{2} m_1 v_{ic} = \frac{1}{2} m_3 v_{fc} \]
due to non change in reaction between d + d and ³He, right and left hand sides of equation will not be equal.

IVa. 

b. X component:
\[ m_1 v_{ic} = m_1 v_{ic} \cos \theta_1 + m_2 v_{fc} \cos \theta_2 \]

y component:
\[ \phi = m_2 v_{fc} \sin \theta_2 - m_1 v_{ic} \sin \theta_1 \]
c. Still have four unknowns $\theta_1$, $\theta_2$, $v_{if}$, and $v_{if}$. Need at least 2 more pieces of information to solve problem... usually conserved kinetic energy (for elastic collisions) and one final reaction parameter (either angle or velocity).

d. Since $m_1 = m_2$

$$v_{ic} = v_{if} \cos \theta_1 + v_{if} \cos \theta_2 \quad \text{conservation of momentum}$$

$$v_{if} \sin \theta_1 = v_{if} \sin \theta_2$$

$$v_{ic}^2 = v_{if}^2 + v_{if}^2 \quad \text{conservation of kinetic energy}$$

by manipulating these equations:

$$v_{if} = v_{ic} \cos \theta_1 = \frac{(500 \text{ m/s}) \cos (60^\circ)}{250 \text{ m/s}}$$

$$v_{if} = \sqrt{v_{ic}^2 - v_{if}^2} = \sqrt{(500 \text{ m/s})^2 - (250 \text{ m/s})^2}$$

$$= \sqrt{187,500 \text{ m}^2/\text{s}^2}$$

$$= 433 \text{ m/s}$$

$$\sin \theta_2 = \frac{v_{if}}{v_{if}} \sin \theta_1$$

$$\sin \theta_2 = \frac{250 \text{ m/s}}{433 \text{ m/s}} \sin 60^\circ = (0.577)(0.8660)$$

$$= 0.50$$

$$\theta_2 = 30^\circ$$
To eliminate $\theta$, square and add first equations

$$\left(\frac{\hbar}{\lambda'} - \frac{\hbar}{\lambda} \cos \varphi\right)^2 = (mv \cos \Theta)^2$$

$$\left(\frac{\hbar}{\lambda'}\right)^2 - \frac{2\hbar^2}{\lambda'^2} \cos \varphi + \left(\frac{\hbar}{\lambda}\right)^2 \cos^2 \varphi = m^2v^2 \cos^2 \Theta$$

$$+ (mv \sin \varphi)^2 = \left(\frac{\hbar}{\lambda} \sin \varphi\right)^2$$

$$\frac{(\hbar)^2}{\lambda'^2} \cos \varphi + \left(\frac{\hbar}{\lambda'}\right)^2 \cos^2 \varphi + \left(\frac{\hbar}{\lambda}\right)^2 \sin^2 \varphi = m^2v^2 \cos^2 \Theta + m^2v^2 \sin^2 \Theta$$

$$\cos^2 \varphi + \sin^2 \varphi = 1$$

$$\left(\frac{\hbar}{\lambda'}\right)^2 = \frac{2\hbar^2}{\lambda'^2} \cos \varphi + \left(\frac{\hbar}{\lambda'}\right)^2 = m^2v^2$$

from energy

$$m^2v^2 = 2m \hbar c \left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

multiply both sides by $\lambda \lambda'$, pull out $\hbar^2$ from left

$$\hbar^2 \left(\frac{\lambda'}{\lambda} - 2\cos \varphi + \frac{\lambda}{\lambda'}\right) = 2m \hbar c \left(\lambda' - \lambda\right)\quad \lambda' - \lambda = \Delta \lambda$$

$$\Delta \lambda = \frac{\hbar}{2mc} \left(\frac{\lambda'}{\lambda} + \frac{\lambda}{\lambda'} - 2\cos \varphi\right)$$

when $\Delta \lambda$ small, $\frac{\lambda'}{\lambda} + \frac{\lambda}{\lambda'} \approx 2$

$$\Delta \lambda = \frac{\hbar}{2mc} (2 - 2\cos \varphi) = \frac{\hbar}{2mc} (1 - \cos \varphi)$$

b. $\Delta \lambda$ maximum when $\cos \varphi = -1$, $\varphi = 180^\circ$

when photon is backscattered to incident photon direction.
\[ a + x \rightarrow y + b \]

**a.**

- In the lab frame: \( p = maV_a \)
- In the CM frame: \( p = 0 = maV'_a + mxV'_x \) where primes indicate center of mass coordinate.

\[ V'_a = V_a - V_{cm} \] where \( V_{cm} \) is center of mass velocity.

\[ V'_x = -V_{cm} \]

Using the CM frame, \( maV'_a = -mxV'_x \), and by substitution:

\[ ma(V_a - V_{cm}) = -mx(-V_{cm}) \]

\[ maV_a - maV_{cm} = +mxV_{cm} \]

\[ maV_a = maV_{cm} + mxV_{cm} \]

\[ V_{cm} = \left( \frac{ma}{ma + mx} \right) V_a \]

**b.** For total momentum to be zero, particles must travel \( 180^\circ \) in CM reference frame.

**c.** For center of mass frame:

\[ m_bV'_b + m_yV'_y = 0 \quad \text{and} \quad V'_b = V_b - V_{cm} \]

\[ V'_y = V_y - V_{cm} \]

\[ m_b(V_b - V_{cm}) + m_y(V_y - V_{cm}) = 0 \]

\[ m_bV_b - m_bV_{cm} + m_yV_y - m_yV_{cm} = 0 \]

\[ m_b V_b + m_y V_y = m_b V_{cm} + m_y V_{cm} \]

\[ \frac{m_bV_b + m_yV_y}{m_b + m_y} = V_{cm} \]

\[ V_{cm} = \left( \frac{maV_a}{ma + mx} \right) \]

Assuming elastic collision: \( m_b + m_y = ma + mx \)

\[ V_{cm,f} = \frac{maV_a}{ma + mx} = V_{cm,i} \]
\[ T_{cm} = 48\,\text{MeV} \]

\[ T_p = T_{cm} \left( \frac{m_t + m_p}{m_p} \right) \]

- \( m_p \): mass of projectile
- \( m_t \): mass of target

\[ T_p = 48\,\text{MeV} \left( \frac{15.9949\,\text{amu} + 63.9280\,\text{amu}}{15.9949\,\text{amu}} \right) = 240\,\text{MeV} \]